Abstract
Systems that exhibit flexible dynamics are susceptible to vibration caused by changes in the reference command, as well as, from external disturbances. Feedback control is an obvious choice to deal with these vibrations, but in many cases, it is insufficient or difficult to implement. Input shaping has proven beneficial for reducing vibration that is caused by changes in the reference command. However, input shaping does not deal with vibration excited by external disturbances. On the other hand, vibration absorbers have long been used to decrease vibration from external sources, particularly sinusoidal disturbances. In this paper, vibration absorbers and input shaping are designed concurrently to reduce vibration from both the reference command and external sources. The usefulness of this combined approach for dealing with both external and reference command disturbances is demonstrated through computer simulations.

1. Introduction
The vibration of flexible systems often limits speed and accuracy. The vibration of such systems is usually caused by changes in the reference command or from external disturbances. If the system dynamics are known, commands can be generated that will cancel the vibration from the system's flexible modes [1-3]. Input shaping is one such command generation scheme that is implemented by convolving a sequence of impulses with the command signal, as shown in Figure 1. The result of the convolution is used as the new reference command for the system. Input shaping has been used in conjunction with a variety of feedback control schemes [4-8]. A block diagram for the case with an input shaper outside the feedback loop and a Proportional-Derivative (PD) feedback controller is shown in Figure 2.

Input shaping has been implemented on numerous systems. The performance of long-reach manipulators [9], cranes [10-15], and coordinate measuring machines [16-18] have been improved with input shaping.

When input shaping is implemented outside of the feedback loop, its vibration-reducing effects do not work on external disturbances. For example, the vibration induced by the disturbances, D and W, shown in Figure 2, would not be reduced by the input shaper. The vibration must be dealt with by the feedback controller. Inserting the input shaping inside the feedback loop creates a time delay within the loop, thereby introducing a stability issue [4, 19].

Another possible solution is to add a vibration absorber to the plant dynamics, Gp [20]. If the primary system can be treated as a mass under PD control, then a model of the augmented system is shown in Figure 3. Vibration absorbers, or mass dampers, cancel unwanted dynamic responses of a system by introducing a new mode of vibration. One of the earliest applications of this principle occurred in 1883 inside the British warship the HMS Inflexible. A volume of water in its hull was used as a secondary oscillating mass [21]. The deck motion resulting from the near sinusoidal forcing of the ocean swells was reduced by the counteracting motion of the water volume.
Another early vibration absorber is described in the patent granted to Frahm in 1911 also dealing with controlling oscillation in ships [22].

The early theoretical work only considered the ideal undamped case [20]. With the advent of computational techniques, it has been possible to design an absorber for a variety of situations and criteria, including damping and non-linearity in both the absorber and primary system [23]. Implementation of multiple absorbers and active absorbers whose parameters are controlled by an external source have also been developed [24].

The use of vibration absorbers has crossed a broad range of applications, including architecture: chimneys, bridges, windmills, communication towers; rotational machinery: pumps, generators, engines; and consumer goods: dishwashers, refrigerators, passenger seating [21, 25]. However, the application to the pre-planned trajectories arising in automated manufacturing has been limited. Traditionally absorbers were designed to cope with periodic, generally sinusoidal excitations, with few excursions into other areas. Absorbers have been designed for random excitations [26], as well as for the "time optimal case" in [27] defined by energy dissipation for an impulse response, but the step-like trajectories followed by many manufacturing and robotics systems have not yet been rigorously explored [28, 29]. This does not mean that vibration absorbers cannot significantly alter the response of a system to the aforementioned step-like disturbances. Figure 4 shows the possible improvement in the step disturbance response of a mass under PD control when a vibration absorber is utilized.

This paper presents a method for concurrently designing vibration absorbers and input shapers to suppress step-like disturbances. This approach provides good vibration reduction for both external and command-induced vibration. Section 2 gives a brief review of input shaping. Section 3 states some of the basic principles of vibration absorber design, while Section 4 describes the process used for designing the vibration absorber and input shaping together. Section 5 demonstrates the effectiveness of combining input shaping with the vibration absorber through simulations. Finally, conclusions are presented in the final section.

2. Review of Input Shaping

The amplitudes and time locations of the impulses in an input shaper are determined by solving a set of constraint equations. Most types of constraints can be categorized as residual vibration constraints, robustness constraints, constraints on the impulse amplitudes, and the requirement of time optimality.

To constrain the residual vibration, we need an expression for the residual vibration amplitude as a function of an impulse sequence. If we assume the system can be modeled as a second-order harmonic oscillator (this analysis can be extended to a superposition of second-order systems) then the system response from a single impulse is [30]:

\[ y_0(t) = \frac{A_0 \omega}{\sqrt{1-\zeta^2}} e^{-\zeta \omega (t-t_0)} \sin (\omega \sqrt{1-\zeta^2} (t-t_0)) \]  

(1)

where \( A_0 \) is the amplitude of the impulse, \( t_0 \) is the time the impulse is applied, \( \omega \) is the natural frequency, and \( \zeta \) is the damping ratio. The response from a sequence of impulses is just a superposition of the response given in (1). The amplitude of this superposition is:

\[ A_\Sigma = \frac{\omega}{\sqrt{1-\zeta^2}} e^{-\zeta \omega n} \sqrt{C(\omega, \zeta)^2 + S(\omega, \zeta)^2} \]  

(2)

where,

\[ C(\omega, \zeta) = \sum_{i=1}^{n} A_i e^{\zeta \omega t_i} \cos (\omega \zeta t_i) \]  

(3)

\[ S(\omega, \zeta) = \sum_{i=1}^{n} A_i e^{\zeta \omega t_i} \sin (\omega \zeta t_i) \]  

(4)

with \( t_i \) as the time of the \( i \)th impulse and \( \omega_i \) is the damped natural frequency. To express the amplitude in a non-dimensional manner, (2) can be divided by the amplitude of residual vibration from a single impulse of unity magnitude. The resulting percentage residual vibration expression gives us the ratio of vibration with input shaping to that without input shaping. The amplitude of residual vibration from a single unity-magnitude impulse is:

\[ A_\uparrow = \frac{\omega}{\sqrt{1-\zeta^2}} \]  

(5)

Dividing (2) by (5) yields the percentage vibration equation:

\[ V(\omega, \zeta) = \frac{A_\Sigma}{A_\uparrow} = e^{-\zeta \omega n} \sqrt{C(\omega, \zeta)^2 + S(\omega, \zeta)^2} \]  

(6)

where \( t_n \) is the time of the final impulse. If \( V(\omega, \zeta) \) is set equal to zero at the modeling parameters, \( (\omega_0, \zeta_m) \), then a shaper that satisfies the equation is called a Zero Vibration (ZV) shaper [2, 3].

A constraint must be applied to ensure that the shaped command produces the same rigid-body motion as the unshaped command. To satisfy this requirement the impulse amplitudes must sum to one:

\[ \sum_{i=1}^{n} A_i = 1 \]  

(7)
Furthermore, the amplitudes of the individual impulses must be limited, or they can go to positive and negative infinity. There are several possible amplitude constraints [15]. Here we limit the impulses to only positive values.

\[ A_i \geq 0 \]  

Due to the transcendental nature of (6), there will be multiple possible solutions. To make the solution time optimal, the shaper duration must be made as short as possible. Therefore, the time optimality requirement is:

\[ \min(t_{du}) \]  

In practice, ZV shapers can be sensitive to modeling errors [31]. To demonstrate this effect, the amplitude of residual vibration can be plotted as a function of a modeling error. The solid line in Figure 5 shows such a sensitivity curve for the ZV shaper. The vertical axis is the percentage vibration. This non-dimensional robustness measure is equal to zero [3]. That is:

\[ 0 = \frac{d}{d\omega}V(\omega, \zeta) \]  

When (6) - (10) are satisfied with \( V = 0 \), the result is a Zero Vibration and Derivative (ZVD) shaper. By comparing the 5% insensitivities shown in Figure 5, it can be concluded that the ZVD shaper is significantly more robust to modeling errors than the ZV shaper.

Other types of robustness constraints have also been proposed but will not be utilized here [32-36]. Any shaped command will have its rise time increased by the duration of the shaper as was shown in Figure 1. Because the duration of the ZVD shaper is twice that of the ZV shaper, the ZVD shaper increases the rise time more than the ZV shaper. This increased rise time is the price that is paid for the increased robustness to modeling errors. With the SI shapers, increasing robustness increases rise time in a nonlinear manner [33].

3. Vibration Absorber Design For Step Disturbances

The design of a vibration absorber to dissipate disturbances requires choosing the correct parameters to convey the energy from the primary mass into the absorber’s vibration. These parameters are the absorber mass \( M_a \), the absorber damper \( C_a \), and the absorber’s spring \( K_a \). It is, however, easier to discuss the frequency and damping ratio of the absorber. Here, the mass is not considered a variable. Instead, it is left to the designer to use the maximum reasonable mass because vibration suppression increases with absorber mass.

In order to choose the best absorber, the response of a fourth-order system has to be quantified. The two primary ways to do this are the peak overshoot of the combined system and the settling time. The peak overshoot of the combined system is not utilized here because the effects of the absorber on this quantity are small [28]. Instead, the settling time is used to determine the best absorber. The settling time is calculated for a fourth-order response through the use of an exponential boundary. The peaks and valleys of the response are fit inside a decaying exponential, with the error minimized. This results in the equation for the boundary of the response, which can be used to calculate any percent settling time. The five percent settling time, or time it takes the response to decay to 5 percent of the move distance and stay there, is used throughout the remainder of the paper.

4. Concurrent Design Of Vibration Absorbers and Input Shapers

In order to design absorbers and the complimentary input shapers at the same time a numerical optimization was performed. The optimization selects both the absorber and shaper parameters while minimizing a cost function. The MATLAB Optimization Toolbox was utilized along with the simulation capabilities of MATLAB and a settling time calculation of the four­th-order response of the combined system. The net result is a solution that decreases vibration from both external and command induced step disturbances. The process proposed here consists of the following steps.  
1) Create a Dynamic Model of the system  
2) Parameterize the Vibration Absorber and Input Shaper  
3) Perform optimization to select parameters while minimizing a cost function.

4.1 Utilizable Variables

Before the optimization can be formulated, the possible solution set has to be defined, i.e. the combination of the variables that the optimization can select. These variables fall into two subcategories, absorber variables, and shaper variables. The absorber variables are the physically realizable characteristics of the absorber namely, the absorber mass, the damping ratio, and the spring
constant. For the input shaper, the variables are the number of impulses and their time locations and magnitudes.

These variables cannot take on arbitrary values; instead, they are limited by both the laws of physics, knowledge gained from previous work, and the desired outcome. These limits form the basis of the constraints and cost functions of the optimization.

4.2 Constraints and Cost Functions

The limits on the optimization fall into three categories, Physical Constraints, Response Constraints, and Cost Function Values.

The Physical Constraints are products of a simulation based environment and are used to limit the options of the optimization algorithm. For example, a negative damping ratio for the absorber is physically unrealizable. The mass of the absorber is also considered a physical constraint. An increase in the mass of the absorber increases its effectiveness at disturbance rejection. A larger mass is able to exert more force on the primary mass and therefore can control the undesired vibration better. Therefore, if absorber mass is considered a variable it will always be driven to towards infinity. The designer must, therefore, choose the appropriate absorber mass for the problem.

The Response Constraints are used as a binary condition for the possible solutions of optimization. If these constraints are satisfied, the current choice of the optimization becomes usable as a solution. These Response Constraints define the desired response of the absorber and input shaping scheme. If a peak overshoot of 2 inches is the maximum permitted for the system, then a response constraint of peak overshoot can be used to limit the possible solutions to only those that have a peak overshoot less than 2 inches. Here, the primary response constraint is the peak overshoot the system. This constraint takes the form:

\[ M_{p,\text{shap}} - M_{p,\text{allow}} < 0 \]  \hspace{1cm} (12)

where \( M_{p,\text{shap}} \) is the peak overshoot of the shaped step response and \( M_{p,\text{allow}} \) is the maximum allowable overshoot.

Finally, the Cost Function Values determine how the optimization is driven towards the solution. The settling time of the combined system subject to an external step-disturbance is the most heavily weighted value in the cost function. This is calculated through an exponential boundary fit of the response. However, the inclusion of the peak overshoot of the augmented system provides stability in the optimization routine. The settling time of the externally disturbed system, \( T_{\text{set}} \), is not affected by any of the shaper variables. Therefore, if a shaper-affected variable is not included, a distinct solution does not exist for the entire scheme. Therefore a cost function of:

\[ J = T_{\text{set}} \times M_{p,\text{shap}} \]  \hspace{1cm} (13)

is used here. \( T_{\text{set}} \) primarily drives the selection of the vibration absorber parameters, while \( M_{p,\text{shap}} \) primarily drives the shaper parameters.

5. Results and Trends

The concurrent design of input shaping and vibration absorbers yields a solution that rejects vibrations caused by both the input to the system and external disturbances. Figure 6 shows an example of this improvement of a system with \( \omega_0=2\pi \) and \( \zeta = 0.1 \) where the settling time from a step disturbance is reduced by 31.4% and the peak overshoot caused by a step change in the reference command is reduced 94.6% thru the inclusion of a vibration absorber with parameters:

\[ k_A = 0.7878 \]
\[ \zeta_A = 0.4000 \]  \hspace{1cm} (14)

and an input shaper given by:

\[ [t_i] = \begin{bmatrix} 0 & 3.3451 \\ A_i \end{bmatrix} \]

\[ A_i = \begin{bmatrix} 0.6051 & 0.3949 \end{bmatrix} \]  \hspace{1cm} (15)

The curves with the square symbols represent the augmented system.

In this example, a two-impulse shaper is used. Using only two impulses forces a fast rise time and simplifies the optimization routine. More impulses or different response parameters can be tuned to better suit the needs of a particular application. For example, if the problem statement required large robustness to modeling errors, then the shaper would need more impulses.

Several trends arise in this concurrent design approach, most notably relating to the allowable peak overshoot. The peak overshoot’s worth as a constraint is related both to its applicability to the problem and its direct relation to the shaper parameters. Lowering the peak overshoot constraint obviously reduces the maximum excursion, as seen in Figure 7. However, more importantly, and unexpectedly, Figure 8 shows that increasing the limit on overshoot allows an improvement in the disturbance rejection properties. Therefore, a designer can trade off directly between disturbance rejection and vibration induced by the reference command.

The shapers designed by this concurrent approach follow some of the same trends seen in standard ZV input shapers. For example, as the system damping ratio increases, the magnitude of the first impulse of the shaper also increases, as seen in Figure 9. This trend occurs due to
the natural damping of the system being able to decay the extra energy placed in the system by an increased impulse.

**Conclusions**

By coupling the command filtering of input shaping and the mechanical effect of a vibration absorber, a scheme can be devised that both rejects external disturbances and eliminates vibration caused by the input to the system. A method was presented for concurrently designing input shapers and vibration absorbers in an optimal manner. Examples showed that substantial improvements in both disturbance rejection and command-induced vibration can be accomplished. With this method, a direct trade off can be made between disturbance rejection and input-forced vibration.

**References**